Calculating The Ex-Post Cost of Capital
Intel - A Case Study

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In this white paper we will calculate the mean and variance of equity returns over time. We want the return mean to be a proxy for the risk-adjusted discount rate (κ) and the return variance to be a proxy for return volatility (σ). To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We will use Intel (INTC) as our sample company equity price time series. The share prices in the table below present an excerpt from the entire time series of Intel share price data (Jan-1985 to Jan-2024)...

Table 1: INTEL Share Prices (Source: Yahoo Finance)

A	**B**	**C**	**D**	**E**	**F**	**G**
Month	Price			Share Prices		
Number	Date	Open	High	Low	Close	Adj Close
0	01/01/85	0.58333	0.67708	0.56771	0.64063	0.36198
–	–	–	–	–	–	
240	01/01/05	23.64000	23.79000	21.89000	22.45000	13.15033
241	02/01/05	22.49000	24.63000	22.17000	23.99000	14.05240
242	03/01/05	24.36000	25.47000	22.96000	23.23000	13.65560
243	04/01/05	23.34000	23.90000	21.94000	23.52000	13.82608
244	05/01/05	23.50000	27.40000	23.35000	26.96000	15.84827
245	06/01/05	26.80000	27.75000	25.73000	26.02000	15.34721
246	07/01/05	26.23000	28.84000	26.08000	27.14000	16.00782
247	08/01/05	27.24000	27.66000	25.31000	25.72000	15.17027
248	09/01/05	25.58000	26.12000	23.80000	24.65000	14.58148
249	10/01/05	24.74000	24.96000	22.75000	23.50000	13.90121
250	11/01/05	23.18000	27.15000	22.53000	26.68000	15.78232
251	12/01/05	26.94000	27.49000	24.87000	24.96000	14.81558
–	–	–	–	–	–	
468	01/01/24	49.20000	50.30000	43.35000	43.65000	43.65000

Note: While the share's closing price (Column F) refers to the actual share price at the end of the trading day, the adjusted closing price (Column G) adjusts the actual closing price for dividends, stock splits, and new stock offerings. Adjusted closing prices assume that dividends are reinvested by purchasing additional shares, and the effects of dilution are eliminated.

Our task is to answer the following questions...

Question 1. Calculate a proxy for the discrete-time discount rate.

Question 2. Calculate a proxy for annual return volatility.

Question 3. How accurate is our cost of capital estimate?

Question 4. Graph the actual return log versus the regression estimate?

Annual Return Mean

We will define one discrete time period to be one month. We will define the variable V_n to be the share's adjusted closing price at the end of discrete-time period n and the variable k to be the periodic cost of capital. The equation for adjusted closing price at time period n given the adjusted closing price at time period $n-1$ is... [1]

$$V_n = V_{n-1}\,(1+k) \tag{1}$$

Using Equation (1) above, the equation for the natural log of adjusted closing price is...

$$\ln(V_n) = \ln(V_{n-1}) + \ln(1+k) \tag{2}$$

Using Equations (1) and (2) above, the equations for adjusted closing price and the log of adjusted closing price at time period n given adjusted closing price and the log of adjusted closing price at time zero are...

$$V_n = V_0\,(1+k)^n \text{ ...such that... } \ln(V_n) = \ln(V_0) + n\,\ln(1+k) \tag{3}$$

Our regression equation using the raw data in Table 1 above is...

$$\ln(V_n) = \alpha + \beta\, n \text{ ...where... } \alpha = \ln(V_0) \text{ ...and... } \beta = \ln\Bigg(1+k\Bigg) \text{ ...and... } n = \text{month number} \tag{4}$$

Using the results of linear regression Equation (4) above, the equation for the discrete-time annualized ex-post cost of capital is...

$$\text{Annual ex-post cost of capital mean} = \text{Exp}\Bigg\{\beta \times 12\Bigg\} - 1 \tag{5}$$

Annual Return Volatility

Using Equation (2) above, the equation for the continuous-time periodic return (i.e. ex-post cost of capital) over the time interval $[n-1, n]$ is...

$$\text{Periodic continuous-time return} = \ln(1+k) = \ln(V_n) - \ln(V_{n-1}) \tag{6}$$

We will define the variable N to be the sample size of our historical share price time series. Using Equation (6) above, the equations for the first moment (FM) and second moment (SM) of our distribution of periodic continuous-time returns are...

$$\text{FM} = \frac{1}{N}\sum_{n=1}^{N}\Bigg[\ln(V_n) - \ln(V_{n-1})\Bigg] \text{ ...and... } \text{SM} = \frac{1}{N}\sum_{n=1}^{N}\Bigg[\ln(V_n) - \ln(V_{n-1})\Bigg]^2 \tag{7}$$

Using Equation (7) above, the equation for the variance of periodic continuous-time returns over the time interval $[n-1, n]$ is...

$$\text{Periodic return variance} = SM - FM^2 \tag{8}$$

Using Equation (8) above, the equation for annual ex-post return volatility is...

$$\text{Annual ex-post cost of capital volatility} = \sqrt{\Bigg(SM - FM^2\Bigg) \times 12} \tag{9}$$

Answers To Our Hypothtical Problem

The raw data table for our linear regression will be constructed as follows...

$$\begin{aligned} X &= \text{Month number (independent variable)} \\ Y &= \text{Natural log of adjusted closing price (dependent variable)} \end{aligned} \tag{10}$$

Column descriptions: A = Month number, B = Log of actual adjusted closing price, C = Change in the log of actual adjusted closing price (i.e. MTD total return), D = Best fit regression estimate of log of adjusted closing price.

Using Table 1 above, the raw data and statistical results for our linear regression per Equation (4) above is...

Table 2: Raw Share Price Data

A	B	C	D
X	Y Act	Y Chg	Y Est
0	-1.01618	–	-0.31095
–	–	–	–
240	2.57645	-0.04102	2.14218
241	2.64279	0.06635	2.15241
242	2.61415	-0.02864	2.16263
243	2.62656	0.01241	2.17285
244	2.76306	0.13650	2.18307
245	2.73093	-0.03213	2.19329
246	2.77308	0.04214	2.20351
247	2.71934	-0.05374	2.21373
248	2.67975	-0.03959	2.22396
249	2.63198	-0.04778	2.23418
250	2.75889	0.12691	2.24440
251	2.69568	-0.06321	2.25462
–	–	–	–
468	3.77620	-0.14081	4.47267

Table 3: Regression Parameters

Description	Columns	Value
Mean of X	A	234
Mean of Y	B	2.08
Variance of X	A	18330
Variance of change in Y	C	0.01144
Covariance of X and Y	A,B	187.36
Correlation of X and Y	A,B	0.89

Using the table data above, the value of the beta parameter in Equation (4) above is... [2]

$$\beta = \text{Covariance of X and Y} \div \text{Variance of X} = \frac{187.36}{18330} = 0.01022 \tag{11}$$

Using the table data above and Equation (11) above, the value of the alpha parameter in Equation (4) above is... [2]

$$\alpha = \text{Mean of Y} - \beta \times \text{Mean of X} = 2.08 - 0.01022 \times 234 = -0.31095 \tag{12}$$

Question 1. Calculate a proxy for the discrete-time discount rate.

Using Equations (5) and (11) above, the answer to the question is...

$$\kappa = \text{Exp}\left\{\beta \times 12\right\} - 1 = 0.01022 \times 12 - 1 = 0.1233 \tag{13}$$

Question 2. Calculate a proxy for annual return volatility.

Using the table data above, the answer to the question is...

$$\sigma = \sqrt{\text{Variance of Chg in Y} \times 12} = \sqrt{0.01144 \times 12} = 0.3705 \tag{14}$$

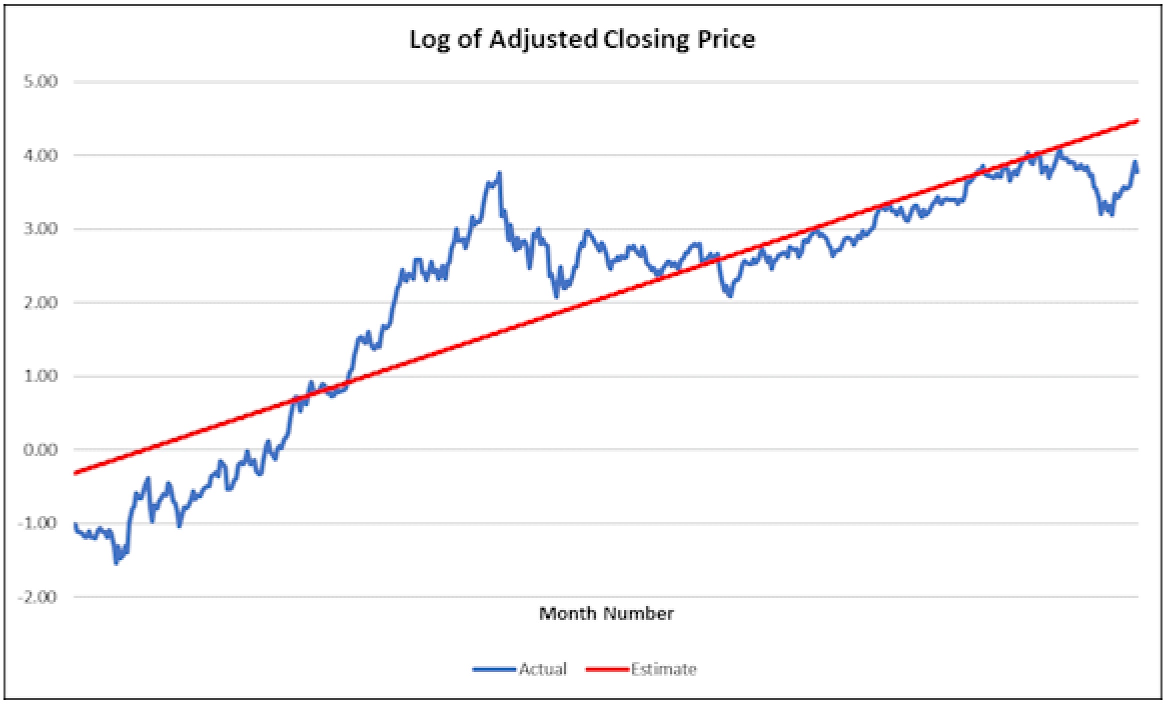
Question 3. How accurate is our cost of capital estimate?

Using the table data above, the r-squared (goodness of fit) value is... [2]

$$R^2 = \left(\text{Correlation of X and Y}\right)^2 = 0.89^2 = 0.79 \tag{15}$$

Question 4. Graph the actual return log versus the regression estimate?

The graph of the column B (actual log of adjusted closing price) and column D (regression estimate of the log of adjusted closing price) is...



Appendix

A. The solution to the following equation is...

$$
\begin{aligned}
S_n &= C_0 \left(1+k-d\right)^n \frac{1+k-d}{1+k} \bigg/ 1 - \frac{1+k-d}{1+k} \\
&= C_0 \left(1+k-d\right)^n \frac{1+k-d}{1+k} \bigg/ \frac{1+k-1-k+d}{1+k} \\
&= C_0 \left(1+k-d\right)^n (1+k-d) \bigg/ d \\
&= C_0 \left(1+k-d\right)^{n+1} \bigg/ d
\end{aligned} \tag{16}
$$

References

[1] Gary Schurman, *Calculating The Ex-Post Cost of Capital - Adjusted Closing Price,* January, 2024.

[2] Gary Schurman, *Univariate Ordinary Least Squares Estimator,* May, 2011.